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MEAN–VaR PORTFOLIO OPTIMIZATION UNDER CAPM WITH LAGGED, NON CONSTANT VOLATILITY AND THE LONG MEMORY EFFECT

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Abstract. In this paper, we discus the method of portfolio optimization based on the mean and the Value-at-Risk (VaR) under the Capital Asset Pricing Model (CAPM) framework with lagged, non-constant volatility and the long memory effect. In CAPM, the returns of individual stocks (or portfolios) are assumed influenced by the market returns and risk-free return. Here, we estimate the stock return betas by extending the CAPM model with lagged market factors, where the market returns are assumed has non-constant volatility, which will be estimated using GARCH models. The long memory effect will be modeled using ARFIMA model. The risk is measured by VaR that is calculated using normal distribution with a confidence level c. Mean and VaR will be used for the formulation of portfolio optimization problems. The portfolio optimization is performed using the Lagrange Multiplier and the solution is obtained by the Kuhn-Tucker theorems. We illustrate these methods using some stocks from the Indonesian capital market.

Keywords: CAPM, ARFIMA models, GARCH models, VaR, Lagrangean Multiplier, Kuhn-Tucker theorem.

1. Introduction

In the real world almost all investments contain an element of uncertainty or risk. Investors do not know with certainty that the results will be obtained from an investment. In such circumstances, it is said that investors are in the investment risk. What he could do in this situation is to estimate the expected return from investments, and measured how far the actual results will deviate from the expected results. The first problem concerned with the calculation of the expected value and the second concerning the measurement of risk values (Elton & Gruber, 1991).

Since the investor faces a risky investment opportunities, investment options can not rely solely on the expected rate of return. If investors expect to earn high rates of return, then he must be willing to bear high risks as well. One of the characteristics of investments in securities is easy to form an investment portfolio. That is, investors can easily spread (diversified) investment in various investment opportunities. Investors to diversify their investments because they want to reduce the risks they bear (Punjerr et al., 1998).

If every individual investors act as we expect, then we will be able to formulate how the investors will behave, and therefore how the price and profit levels will be determined by the market. Formation models of general equilibrium allows us to determine the relevant measure of risk and how the relationship between the risk for each asset if the capital markets are in balance. One of models is the Capital Asset Pricing Model (CAPM). If the CAPM is the influence and the relationship between the variable values that have occurred in a period with what happened in the next period, then this model is called a CAPM with a delay (Jag) (Cole et al., 2007).

In the CAPM, the risk factors are described by a value called a beta \( \beta_f \), and expected rate of return of a stock is equal to the level of risk-free returns \( \mu_f \), plus the risk premium is the difference between market gains \( R_m \), with the risk-free profits \( \mu_f \), multiplied by beta \( \beta_f \). The greater the stock's risk (i.e. beta), the greater the expected rate of return for the stock (Firmandez, 2002; Sukono, Subanan & Dedi, 2009). In reality, the market benefits are often highly fluctuating, could have a volatility that is not constant, and also have a long memory effect (Kang & Yoon, 2007; Tsay, 2005).

In this paper we analyze the market returns using the time series model, by considering a long memory effect which is modeled using ARFIMA model and where the volatility is assumed non-constant and is modeled using GARCH models. Meanwhile, the risk will be measured using the Value-at-Risk (VaR) model (Elton & Gruber, 1991). This paper aims to investigate the proportion of capital allocation which will be invested in each
stock portfolio, so that will be gained the maximum return portfolio and the minimum level of risk (in terms of VaR). Optimization is done by using the Lagrangean multiplier and the Kuhn-Tucker theorems. Furthermore, we provide some empirical results using some stocks price data that are traded in the Indonesian capital market.

2. Methodologies

2.1 Time Series Models

Mean Model. Let \( \eta_t \) is a log return of asset at time \( t \). There exist some time series models whose autocorrelation function (ACF) decays slowly to zero at a polynomial rate as the lag increases. These processes are referred to as long-memory time series. One such example is the fractionally differenced process defined by

\[
(1 - B)^d \eta_t = \alpha_t; \quad -0.5 < d < 0.5
\]

where \( \{\alpha_t\} \) is a white noise series. Properties of model (1) have been widely studied in the literature (Tsay, 2005; Yu & So, 2004). We summarize some of these properties below.

1) If \( d < 0.5 \), then \( \eta_t \) is a weakly stationary process and has the infinite MA representation

\[
\eta_t = \alpha_t + \sum_{i=1}^{\infty} \psi_i \alpha_{t-i}, \text{ with } \psi_k = \frac{d(1+d)...(k-1+d)}{k!} \cdot \frac{(k+d-1)!}{(d-1)!}.
\]

2) If \( d > -0.5 \), then \( \eta_t \) is invertible and has the infinite AR representation

\[
\eta_t = \sum_{i=1}^{\infty} \pi_i \alpha_{t-i} \alpha_t + \sum_{i=1}^{\infty} \pi_i \alpha_{t-i}, \text{ with } \pi_k = \frac{-d(1-d)...(k-1-d)}{k!} \cdot \frac{(k-d-1)!}{(d-1)!}.
\]

3) For \(-0.5 < d < 0.5\), the ACF of \( \eta_t \) is

\[
\rho_k = \frac{d(1+d)...(k-1+d)}{(1-d)(2-d)...(k-d)} \cdot \frac{k!}{(d-1)!}, \quad k = 1, 2, ....
\]

In particular, \( \rho_1 = d/(1-d) \) and \( \rho_k \approx (-d)k^{2d-1}/(d-1)! \), as \( k \to \infty \).

4) For \(-0.5 < d < 0.5\), the partial autocorrelation function (PACF) of \( \eta_t \) is \( \phi_{k,k} = d/(k-d) \), for \( k = 1, 2, .... \)

5) For \(-0.5 < d < 0.5\), the spectral density function \( f(\omega) \) of \( \eta_t \), which is the Fourier transform of the ACF of \( \eta_t \), satisfies

\[
f(\omega) \sim \omega^{-2d}, \text{ as } \omega \to \infty.
\]

where \( \omega \in [0, 2\pi] \) denotes the frequency.

For integer power, using the binomial theorem we obtain

\[
(1-d)^d = \sum_{k=0}^{\infty} (-1)^k \left(\begin{array}{c} d \\ k \end{array}\right) B^k, \quad \left(\begin{array}{c} d \\ k \end{array}\right) = \frac{d(d-1)...(d-k+1)}{k!}.
\]

If the fractionally differenced series \((1-B)^d \eta_t\) follow an ARMA\((p,q)\) model, then \( \eta_t \) is called an ARFIMA\((p,d,q)\) process, which is a generalized ARIMA model by allowing for non integer \( d \) (Kang & Yoon, 2007; Tsay, 2005; Yu & So, 2004).

Variance model. Bollerslev (1986) proposes a model which is called as the generalized autoregressive condition heteroscedastic (GARCH) model. For a log return series \( \eta_t \), let \( \alpha_t = \eta_t - \mu_t \) be the innovation at a time \( t \). Then \( \alpha_t \) follow a GARCH\((p,q)\) model if

\[
\alpha_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\]

where \( \{\epsilon_t\} \) is a sequence of iid random variables with mean 0 and variance 1, \( \alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, \) and

\[
\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1 \quad (Sukono, Subanar & Dedi, 2006; Tsay, 2005; Yu & So, 2004).
\]

The properties of GARCH models can easily be seen by analyzing the simplest GARCH\((1,1)\) model

\[
\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad 0 \leq \alpha_1, \beta_1 \leq 1, \quad (\alpha_1 + \beta_1) < 1.
\]
First, a large \( \alpha_{t-1}^2 \) or \( \alpha_{t-1}^2 \) gives rise to large \( \sigma_t^2 \). This means that a large \( \alpha_{t-1}^2 \) tends to be followed by another large \( \alpha_t^2 \), generating the well-known behavior of volatility clustering in financial time series. Second, it can be shown that if \((1-2\alpha_t^2-(\alpha_t+\beta_t)^2)>0\), then

\[
\frac{E(\alpha_t^4)}{[E(\alpha_t^2)]^2} = \frac{3[1-(\alpha_t+\beta_t)^2]}{1-(\alpha_t+\beta_t)^2-2\alpha_t^2} > 3.
\]

**Kurtosis of GARCH models.** The excess kurtosis of \( \alpha_t \), if it exists, is equal to

\[
K_\alpha = \frac{E(\alpha_t^4)}{[E(\alpha_t^2)]^2} - 3 = \frac{(K_\varepsilon + 3)[1-(\alpha_t+\beta_t)^2]}{1-2\alpha_t^2-(\alpha_t+\beta_t)^2-K_\varepsilon\alpha_t^2} - 3.
\]

(5)

First, consider the case that \( \varepsilon_t \) is normally distributed. In this case, \( K_\varepsilon = 0 \) and using some algebra it can be shown that

\[
K_\alpha^{(g)} = 6\alpha_t^2/[1-2\alpha_t^2-(\alpha_t+\beta_t)^2],
\]

where the superscript \((g)\) is used to denote Gaussian distribution. Second, consider the case that \( \varepsilon_t \) is a (standardized) Student-\( t \) distribution with \( \nu \) degrees of freedom, we have

\[
E(\varepsilon_t^4) = 6/(\nu-4) + 3 \text{ if } \nu > 4.
\]

Therefore, the excess kurtosis of \( \varepsilon_t \) is \( K_\varepsilon = 6/(\nu-4) \) for \( \nu > 4 \). The excess kurtosis of \( \alpha_t \) becomes

\[
K_\alpha = \frac{6+(\nu+1)K_\varepsilon^{(g)}}{\nu-4-K_\varepsilon^{(g)}}.
\]

(6)

Provide that \((1-2\alpha_t^2(\nu-1)/(\nu-4)-(\alpha_t+\beta_t)^2)>0\) (Shi-Jie Deng, 2004; Tsay, 2005).

**2.2 CAPM with Lagged and VaR**

Let \( R_{it} \) be the return at time \( t \) on the \( i \)th asset. Similarly, let \( R_{mi} \) and \( \mu_f \) be the return on the market portfolio and the risk-free return at time \( t \). The security characteristic line (sometimes shortened to the characteristic line) is a regression model:

\[
R_{it} = \mu_f + \beta_i(R_{mt} - \mu_f) + \varepsilon_{it}
\]

(7)

Where \( \varepsilon_{it} \) is \( N(0,\sigma_{\varepsilon t}^2) \). It is often assumed that the \( \varepsilon_{it} \)s is uncorrelated across assets, that is, \( \varepsilon_{it} \) is uncorrelated with \( \varepsilon_{jt} \) for \( i \neq j \) (Fernandez, 2002; Panjer et al., 1998; Sukono, Subanar & Dedi, 2009).

Equation (7) can be extended into the CAPM model with lagged market factors as follow:

\[
R_{it} - \mu_f = \alpha_i + \beta_{i1}(R_{mt} - \mu_f) + \beta_{i2}(R_{mt-1} - \mu_f-1) + \ldots + \beta_{is}(R_{mt-s} - \mu_f-s) + \varepsilon_{it}
\]

(8)

The beta coefficients can be estimated by least square method. Where \( \alpha_i \) is fixed return of \( i \)th asset, and \( s \) is time of lag (Cole et al., 2007; Sukono, Subanar & Dedi, 2009). When, the risk-free return is constant relatively to mean \( \mu_f \), the equation (8) can be rewritten as

\[
R_{it} - \mu_f = \alpha_i + \sum_{l=0}^{s} \beta_{il}(R_{mt-l} - \mu_f-l) + \varepsilon_{it}
\]

(9)

Let \( \mu_{it} = E(R_{it}) \) and \( \mu_{mt} = E(R_{mt}) \). Taking expectation in (9) we have

\[
\mu_{it} = \alpha_i + \mu_f + \sum_{l=0}^{s} \beta_{il}(\mu_{mt-l} - \mu_f).
\]

(10)

If it is assumed that the \( E((R_{it-s} - \mu_f)(R_{jt-u} - \mu_f)) = 0 \) for \( s \neq u \), \( E[\varepsilon_{it}(R_{it-s} - \mu_f)] = 0 \) and \( E(\varepsilon_{it}^2) = \sigma_{\varepsilon t}^2 \), then the variance of (9) is
\[ \sigma^2_{it} = \sum_{l=0}^{S} \beta_l^2 \sigma^2_{mt-l} + \sigma^2_{ei} \]  

By the time series model, for \( \zeta \)-step ahead forecast, we can rewrite that

\[ \mu_{it} = \alpha_i + \mu_f + \sum_{l=0}^{S} \beta_l (\mu_{mt-l}(\zeta) - \mu_f), \]

and

\[ \sigma^2_{it} = \sum_{l=0}^{S} \beta_l^2 \sigma^2_{mt-l}(\zeta) + \sigma^2_{ei}. \]

The Value-at-Risk (VaR) for the individual \( i \)th asset can be calculated as follows:

\[ VaR_{it} = z_{\delta} \hat{\sigma}_{it} = z_{\delta} \left( \sum_{l=0}^{S} \beta_l^2 \sigma^2_{mt-l}(\zeta) + \sigma^2_{ei} \right)^{1/2} - (\alpha_i + \mu_f + \sum_{l=0}^{S} \beta_l (\mu_{mt-l}(\zeta) - \mu_f)), \]

where \( z_{\delta} \) is the percentile of the standard normal distribution when given confidence level \( \delta \) (Khindanova & Rachev, 2005; Sukono, Subanar & Dedii, 2009; Tsay, 2005).

For the portfolio investment, portfolio return by the weight \( w^T = (w_1, ..., w_N)^T \) with \( \sum_{i=1}^{N} w_i = 1 \) is defined as \( R_w = \sum_{i=1}^{N} w_i R_{it} \). Hence, we have the mean is \( \mu_w = \sum_{i=1}^{N} w_i \mu_i \) and the variance is \( \sigma^2_w = \sum_{i=1}^{N} w_i^2 \sigma^2_i + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \sigma_{ij} w_j \) for \( i \neq j \), where the covariance is \( \sigma_{ij} = \text{Cov}(R_{it}, R_{jt}) = E[(R_{it} - \mu_i)(R_{jt} - \mu_j)] \), so that \( \sigma_w = \sqrt{\sigma^2_w} \). The Value-at-Risk (VaR) for portfolio is \( VaR_{w} = z_{\delta} \sigma_w - \mu_w \) (Khindanova & Rachev, 2005; Sukono, Subanar & Dedii, 2009; Yu & So, 2004).

### 2.3 Portfolio Optimization

We assume that the vector of expected values and the covariance matrix are given by \( \mu = (\mu_1, ..., \mu_N) \), with \( \mu_i = E[R_i] \), \( i = 1, ..., N \) and \( \Sigma = (\sigma_{ij})_{i,j=1, ..., N} \), with \( \sigma_{ij} = \text{Cov}(R_i, R_j) \), \( i, j = 1, ..., N \). As mentioned before, the weight of return on a portfolio \( w^T = (w_1, ..., w_N)^T \), where \( \sum_{i=1}^{N} w_i = 1 \) or \( w^T w = 1 \) with \( e = (1, 1, ..., 1)^T \) is as a vector size unit. According to the mean and the variance formulas for portfolio, we obtain \( \mu_w = E[R_w] = \mu^T w \), \( \sigma^2_w = \text{Var}(R_w) = w^T \Sigma w \) and \( \text{Var}R_w = z_{\delta} \sigma_w - \mu_w = z_{\delta}(w^T \Sigma w)^{1/2} - \mu^T w \).

A portfolio \( w^* \) is called (mean-VaR) efficient if there exists no portfolio \( w \) with \( \mu_w \geq \mu_{w^*} \) and \( VaR_{w} < VaR_{w^*} \) (Panjer et al., 1998; Sukono, Subanar & Dedii, 2009). To get efficient portfolios, practitioners use a very simple type of objective function, i.e. to maximize \( 2r \mu_{w^*} - VaR_{w^*} \), \( r \geq 0 \) where \( r \) is the risk tolerance of the investors, \( VaR_{w^*} \) denotes the Value-at-Risk of portfolio, calculated as in section 2.2. Hence, for an investor with risk tolerance \( r \geq 0 \) we must solve the optimization problem (Elton & Gruber, 1991; Panjer et al., 1998; Sukono, Subanar & Dedii, 2009; Yoshimoto, 1996):

\[ \max \{2r \mu^T w - z_{\delta}(w^T \Sigma w)^{1/2} + \mu^T w \} \]

subject to \( w^T w = 1 \)

Where \( e = (1, 1, ..., 1)^T \in \mathbb{R}^N \) as a vector size unit. Since any covariance matrix \( \Sigma \) is positive semi-definite, the objective function is quadratic concave. Hence, (10) is a quadratic concave optimization problem. Its Lagrangian function is given by \( L(w, \lambda) = (2r + 1) \mu^T w - z_{\delta}(w^T \Sigma w)^{1/2} + \lambda (e^T w - 1) \). Because of the Kuhn-Tucker theorem, the optimality conditions are \( \partial L / \partial w = (2r + 1) \mu - z_{\delta} \Sigma w / (w^T \Sigma w)^{1/2} + \lambda e = 0 \) and \( \partial L / \partial \lambda = e^T w - 1 = 0 \).
For \( \tau = 0 \), we have a minimum VaR portfolio \( \mathbf{w}^{\text{Min}} \). Based on algebra calculations and we take the values: \( A = e^T \Sigma^{-1} e \), \( B = \mu^T \Sigma^{-1} e + e^T \Sigma^{-1} \mu \) and \( C = \mu^T \Sigma^{-1} \mu - \frac{1}{\delta} \), we have

\[
\lambda = \frac{-B + (B^2 - 4AC)^{1/2}}{2A}
\]

(16)

By conditional \( (B^2 - 4AC) \geq 0 \) (Panjer et al., Sukono, Subanar & Dedi, 2009)

\[
\mathbf{w}^{\text{Min}} = \frac{\Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e}
\]

(17)

For \( \tau > 0 \), we have an optimum portfolio \( \mathbf{w}^* \). Based on some algebra and we take the values: \( A = e^T \Sigma^{-1} e \), \( B = (2\tau + 1)(\mu^T \Sigma^{-1} e + e^T \Sigma^{-1} \mu) \) and \( C = (2\tau + 1)^2 (\mu^T \Sigma^{-1} \mu) - \frac{1}{\delta} \), so the multiplier \( \lambda \) can be calculated by equation (16), and we have

\[
\mathbf{w}^* = \frac{(2\tau + 1) \Sigma^{-1} \mu + \lambda \Sigma^{-1} e}{(2\tau + 1)e^T \Sigma^{-1} \mu + \lambda e^T \Sigma^{-1} e}
\]

(18)

If vector \( \mathbf{w}^{\text{Min}} \) is substituted into \( \mu_w = E[R_w] = \mu^T \mathbf{w} \) and VaR \( \mathbf{w} = \frac{\delta}{\mathbf{w}^T \Sigma \mathbf{w}} \), then we have the minimum expected return portfolio with a minimum Value-at-Risk. Similarly, for vector \( \mathbf{w}^* \), then we have the optimum expected return portfolio (Panjer et al., 1998; Yoshimoto, 1996).

3. Numerical Results
To illustrate the application of the methods described above, we use it to analyze the stock return of the five companies in Indonesia using stock transaction data from January 26th, 2006 to June 4th, 2009, which is accessed through the Internet. The names of the company are expressed in the form of code alphabets, i.e., \( S_1, S_2, S_3, S_4 \) and \( S_5 \). For the proxy of the market index data, we use IHSG data (\( R_{mt} \)) and for the risk-free asset return we use the average value of the obligation return which the amount is \( \mu_f = 0.090267 \).

3.1 Estimation of \( \alpha_i \) Constants and \( \beta_{si} \) Parameters
Based on the return data of the five companies, we calculate the constants \( \alpha_i \) and the coefficients \( \beta_{si} \) by regression them with the market return. From this step, we also obtain the residual variances of the respective stocks. For the regression analysis, we also apply ANOVA and the determination coefficients, but here we only show \( t \)-statistic (written under of \( \alpha_i \) constants and \( \beta_{si} \) coefficients), statistic of R-Square (R-Sq.) and \( F \)-statistic. Here, we assume only the third lags in the CAPM model is significant in the excess returns equation (8) and denoted as \( I_{t-5} = (R_{mt-5} - \mu_f) ; s = 0, 1, 2, 3 \). The results are summarized in Table-1 below.

<table>
<thead>
<tr>
<th>Stocks</th>
<th>Regression Models</th>
<th>R-Sq. (%)</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>( S_{1t} = 0.0396 + 0.0654 I_t + 0.0367 I_{t-1} + 0.0283 I_{t-2} + 0.0348 I_{t-3} + e_{1t} )</td>
<td>98.78</td>
<td>2.29687E+08</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>( S_{2t} = 0.0578 + 0.0824 I_t + 0.0343 I_{t-1} + 0.0515 I_{t-2} + 0.0436 I_{t-3} + e_{2t} )</td>
<td>99.85</td>
<td>7.77050E+08</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>( S_{3t} = 0.0687 + 0.1040 I_t + 0.1160 I_{t-1} + 0.0577 I_{t-2} + 0.0362 I_{t-3} + e_{3t} )</td>
<td>99.99</td>
<td>9.61894E+08</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>( S_{4t} = 0.0765 + 0.0455 I_t + 0.0719 I_{t-1} + 0.0120 I_{t-2} + 0.0151 I_{t-3} + e_{4t} )</td>
<td>98.67</td>
<td>3.75703E+08</td>
</tr>
<tr>
<td>( S_5 )</td>
<td>( S_{5t} = -0.0599 + 0.0436 I_t + 0.0253 I_{t-1} + 0.0752 I_{t-2} + 0.0261 I_{t-3} + e_{5t} )</td>
<td>99.34</td>
<td>1.47577E+08</td>
</tr>
</tbody>
</table>

Based on the hypothesis of \( H_0 : \beta_{si} = 0 \) versus \( H_1 : \beta_{si} \neq 0 \), from Table-1 shown that by significance level \( \alpha = 0.05 \) statistic \( |t_{si}| \) the values are more than \( |t_{1/2}| = 1.92 \), therefore the hypothesis of \( H_0 \) rejected,
meaning \( \beta_{d1} \neq 0 \) is significant. We also obtain that the values of R-Square (R-Sq) almost close to 100% which indicate both response and predictor variables correlated strongly. Furthermore, statistical values of \( F \), with significance level \( \alpha = 0.05 \) are significantly large than \( F(0.05,3,834) = 2.60 \), meaning the whole models are meaningful.

We further test the normality of the residuals \( \epsilon_{it} \) from the models considered in Table-1, and the results are summarized in Table-2:

<table>
<thead>
<tr>
<th>Residual</th>
<th>Distributions</th>
<th>AD</th>
<th>Prob.</th>
<th>( \mu_{ei} )</th>
<th>( \sigma_{ei} )</th>
<th>( \sigma_{ei}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_{1t} )</td>
<td>Normal</td>
<td>0.141</td>
<td>0.973</td>
<td>0.0004586</td>
<td>0.07594</td>
<td>0.005767</td>
</tr>
<tr>
<td>( \epsilon_{2t} )</td>
<td>Normal</td>
<td>0.238</td>
<td>0.781</td>
<td>-0.0000515</td>
<td>0.05796</td>
<td>0.003359</td>
</tr>
<tr>
<td>( \epsilon_{3t} )</td>
<td>Normal</td>
<td>0.423</td>
<td>0.320</td>
<td>-0.0003540</td>
<td>0.07077</td>
<td>0.005008</td>
</tr>
<tr>
<td>( \epsilon_{4t} )</td>
<td>Normal</td>
<td>0.272</td>
<td>0.669</td>
<td>-0.0002039</td>
<td>0.071351</td>
<td>0.005091</td>
</tr>
<tr>
<td>( \epsilon_{5t} )</td>
<td>Normal</td>
<td>0.344</td>
<td>0.487</td>
<td>-0.0002142</td>
<td>0.04724</td>
<td>0.002232</td>
</tr>
</tbody>
</table>

From Table-2, it appears that all the residuals \( \epsilon_{it} \) are normally distributed with significance level \( \alpha = 0.05 \). The estimated values of the mean parameter is \( \mu_{ei} \) is close to zero, and the estimated of variance is given in the column \( \sigma_{ei}^2 \). Thus the regression models in Table-1 can be used for further analysis.

3.2 Estimations of Mean and Variance Models of Market Return

In the analysis, the Composite Stock Price Index (IHSG) is used as the proxy for the market values. We calculated the log returns of this index and in the following we analyzed the long memory effect, the models for mean and the variance.

Identification of Long Memory Effect

To identify the existence of the long memory effects, We estimate the fractional difference parameter \( d \) in equation (1). For determining of the value of the fractional difference \( d \), we use the method of Geweke and Porter-Hudak, implemented in the \( R \) software. We obtain the fractional value of the difference \( \hat{d} = 0.3613183 \) and the standard error \( SE(d) = 0.1462239 \). To further ensure the existence of the pattern of long memory, we test the hypothesis \( H_0 : \hat{d} = 0 \) versus \( H_1 : \hat{d} \neq 0 \). Based on our calculation, we obtain statistic \( Z = 5.86 \), whereas for a significance level of \( \alpha = 0.95 \) from the table of the standard normal the value obtained \( Z_{0.95} = 1.645 \). Because the value of \( Z \) is greater than the value of \( Z_{0.95} \), we conclude that the test is significant, meaning the market return data following the pattern of long memory. The 95% of confidence interval for the fractional difference parameters \( \hat{d} \) are determined based on the formula \( \hat{d} - Z_{0.025}SE(d) < \hat{d} < \hat{d} + Z_{0.025}SE(d) \), and the result is \( 0.074719 < \hat{d} < 0.647917 \). Because \( \hat{d} \) is within the interval \( 0 < \hat{d} < 1 \), we conclude \( \hat{d} \) can be trusted. In the next step we will apply the fractional difference constant \( \hat{d} = 0.3613183 \) to estimate the mean and the variance models to the market returns data.

Estimation of Mean Model

In this section we used the Eviews 4 software. In the first step of the analysis, we identify and estimate the model that suitable for the market return data. Identification of the model is carried out through the sample of autocorrelation function (ACF) and partial autocorrelation function (PACF) of the fractional differenced data. From the correlogram of the market return (Figure-1), it can be shown that the ACF drastically decreases after the lag 1. While the patterns of PACF have some spikes after lag 1. Based on the ACF and PACF patterns, we consider the possible models for the market return data are the AR(1), MA(1) and ARMA(1,1) models. It can be shown that the best model is the ARMA(1,1) model \( r_{mt} = 0.239800r_{mt-1} + 0.997450u_{t-1} + u_t \), or the ARFIMA(1,d,1) model (where \( \hat{d} = 0.3613183 \)) \( (1 - 0.239800B)(1 - B)^{0.3613183}r_{mt} = (1 + 0.997450B)u_t \).
In the second step of the analysis, we do the diagnostic check for ARMA(1,1) model. Diagnostic test is conducted using the corregogram of residuals and a Ljung-Box hypothesis test. We obtain that the residuals of the ARMA(1,1) model are white noise. We further test the normality of the residual \( \eta_t \). Test results showed that the residual \( \eta_t \) is normally distributed. So we do not need to look for alternative models.

**Estimation of Variance Model**

In this section we used the Eviews 4 software. In the first step, we detect existence of autoregressive conditional heteroscedasticity (ARCH) component of the residual squares of ARMA(1,1) model using the ARCH-LM method. The results showed that the calculated values of \( \chi^2 \) (obs * R-Square) is 3.921869 with probability 0.0000 or \( \alpha \) less than 5% which means that the existence of ARCH components. The same conclusion can be obtained using the correlogram of the squared of residuals.

In the second step, we identify and estimate the variance model. Variance model will use the model of generalized autoregressive conditional heteroscedasticity (GARCH) described by equation (3). Based on square residual correlogram, the ACF graph falls gradually after lag 1, whereas the PACF graph falls drastically after lag 1. This we can estimate tentatively a GARCH(1,1), GARCH(1,1)-M and GARCH(2,2) models. After multiple times of the estimation process, finally we obtain the best estimates is the GARCH(1,1) model, as shown in Table-3. We obtain the best variance model is GARCH(1,1) described by the equation

\[
\sigma_{\eta_t}^2 = 1.04E^{-08} + 0.077409\sigma_{\eta_{t-1}}^2 + 0.886862\sigma_{\eta_{t-1}}^2 + \nu_t.
\]

It can be shown that based on ARCH-LM test, the GARCH(1,1) model also fulfill the diagnostic test.

**Table-3: Estimation of Variance Model**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.305-08</td>
<td>0.015-07</td>
<td>0.8000</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.073579</td>
<td>0.003418</td>
<td>0.0020</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.900528</td>
<td>0.006367</td>
<td>0.5000</td>
</tr>
</tbody>
</table>

**Dependent Variable:** MARKET_RETUR\_JRM

**Method:** ML - ARCH (Marquardt)

**Date:** 1/16/0000  Time: 21:21

**Included observatory:** 890 after adjusting end points

**Convergence achieved after 45 iterations**

**MA Autocorrelation 1. Variance backward 0.**
3.3 Portfolio Optimization Analysis

Using the estimation results in section 3.1 and section 3.2, the next stages, we perform portfolio optimization, as follows. In this section we used the Matlab 7.0 software.

**Predicting the Mean and Variance**

From previous section, we obtain the mean and variance models are as follows:

\[ \hat{\mu}_{mt} = 0.239890 \mu_{mt-1} - 0.997450 \mu_{t-1} \]

and

\[ \hat{\sigma}^2_{mt} = 1.04E - 0.08 + 0.077409 \hat{\sigma}^2_{mt-1} + 0.886862 \sigma^2_{mt-1} \]

We notice that \( \mu_{lt-s} = \hat{\mu}_{mt-s} (1) - \mu_f \); \( s = 0,1,2,3 \). Using the forecast result of 1-step ahead of mean model above it can be calculated the statistical values of \( \mu_{lt} = \hat{\mu}_{mt} (1) - \mu_f = 0.0265810 \), \( \mu_{lt-1} = \hat{\mu}_{mt-1} (1) - \mu_f = 0.0210836 \), \( \mu_{lt-2} = \hat{\mu}_{mt-2} (1) - \mu_f = 0.0038183 \) and \( \mu_{lt-3} = \hat{\mu}_{mt-3} (1) - \mu_f = 0.0382866 \). Where the forecast of 1-step ahead of variance model can be calculated as

\[ \sigma^2_{lt} = \hat{\sigma}^2_{mt} (1) = 0.0010575 \], \( \sigma^2_{lt-1} = \hat{\sigma}^2_{mt-1} (1) = 0.0015885 \), \( \sigma^2_{lt-2} = \hat{\sigma}^2_{mt-2} (1) = 0.00126788 \) and \( \sigma^2_{lt-3} = \hat{\sigma}^2_{mt-3} (1) = 0.00291462 \). Furthermore, using the values of regression of \( \beta_{si} \) from Table-1, we obtain the values of \( \mu_{lt-s}; s = 0,1,2,3 \) and by equation (10) it can be obtained the vector

\[ \mu_s = (\mu_{s1}, \mu_{s2}, \mu_{s3}, \mu_{s4})^T = (0.0454, 0.0657, 0.0788, 0.0810, 0.0653)^T \]

Also using the values of \( \beta_{si} \), we obtain the values of \( \sigma^2_{lt-s}; s = 0,1,2,3 \) and by equation (11) we obtain the variance vector

\[ \sigma^2_s = (\sigma^2_{s1}, \sigma^2_{s2}, \sigma^2_{s3}, \sigma^2_{s4})^T = (0.0058, 0.0034, 0.0050, 0.0051, 0.0022)^T \]

With the assumption that there are no cross correlation between stocks, then \( \text{Corr}(S_i, S_j) = 0; i \neq j \) we obtain

\[
\Sigma = \begin{bmatrix}
0.0058 & 0 & 0 & 0 & 0 \\
0 & 0.0034 & 0 & 0 & 0 \\
0 & 0 & 0.0050 & 0 & 0 \\
0 & 0 & 0 & 0.0051 & 0 \\
0 & 0 & 0 & 0 & 0.0022
\end{bmatrix}
\]

and

\[
\Sigma^{-1} = \begin{bmatrix}
172.41 & 0 & 0 & 0 & 0 \\
0 & 294.12 & 0 & 0 & 0 \\
0 & 0 & 200.00 & 0 & 0 \\
0 & 0 & 0 & 196.08 & 0 \\
0 & 0 & 0 & 0 & 454.55
\end{bmatrix}
\]

**Calculating the Optimum Weight of Portfolio**

From the above results, we consider the optimization problem using (15). The summary of the optimization results is given as follows:

- For risk tolerance \( \tau = 0 \), using equation (16) we obtained multiplier \( \lambda = -0.0231 \). Furthermore, by using equation (17), obtain the minimum weight vector is \( w_{\text{Min}} = (0.0664, 0.2159, 0.1918, 0.1957, 0.3303)^T \). If the minimum weight vector is substituted into the equation \( \mu_w = \sum_{s=1}^{5} w_s \mu_{si} \), we obtain the minimum mean of portfolio return is \( \mu_w = 0.0697 \), and substituted into the equation \( Var_w = z^2 \mu_w (w^T \Sigma w)^{1/2} - \mu^T w \)

with the significance level \( \delta = 0.95 \), we obtain the \( Var_w = 0.0466 \). This means that to get the expected return of 0.0697 units with value-at-risk 0.0466, then the funding for 1 unit must be allocated for 6.64% in stock \( S_1 \), 21.59% in stock \( S_2 \), 19.18% in stock \( S_3 \), 19.57% in stock \( S_4 \) and 33.03% in stock \( S_5 \).

- For risk tolerance \( \tau = 0.4 \), using equation (16) we obtained multiplier \( \lambda = -0.0798 \). Furthermore, by using equation (18) obtain the maximum weight vector is \( w_{\text{Max}} = (0.0064, 0.2089, 0.2290, 0.2392, 0.3165)^T \). If the maximum weight is substituted into the equation \( \mu_w = \sum_{s=1}^{5} w_s \mu_{si} \), we obtain the maximum mean of portfolio return is \( \mu_w = 0.0721 \), and substituted into the equation \( Var_w = z^2 \mu_w (w^T \Sigma w)^{1/2} - \mu^T w \)

with the significance level \( \delta = 0.95 \), we obtain the \( Var_w = 0.0500 \). This means that to get the expected return of 0.0721 units with value-at-risk 0.0500, then the funding for 1 unit must be allocated for 6.4% in stock \( S_1 \), 20.89% in stock \( S_2 \), 22.90% in stock \( S_3 \), 23.92% in stock \( S_4 \) and 3% in stock \( S_5 \).

- The risk tolerances \( \tau > 0.4 \) are not feasible investments. For some feasible risk tolerances, the results are given in Table-4 below.
Table 4: Feasible Portfolios Investment

<table>
<thead>
<tr>
<th>( r )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( w_4 )</th>
<th>( w_5 )</th>
<th>( \mu_y )</th>
<th>( VaR_y )</th>
<th>( \text{Value} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0564</td>
<td>0.2159</td>
<td>0.1918</td>
<td>0.1957</td>
<td>0.3303</td>
<td>0.0567</td>
<td>0.0486</td>
<td>Minimum</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0555</td>
<td>0.2151</td>
<td>0.1951</td>
<td>0.2067</td>
<td>0.3227</td>
<td>0.0700</td>
<td>0.0489</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.0525</td>
<td>0.2143</td>
<td>0.2004</td>
<td>0.2057</td>
<td>0.3271</td>
<td>0.0702</td>
<td>0.0472</td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.0483</td>
<td>0.2134</td>
<td>0.2046</td>
<td>0.2110</td>
<td>0.3254</td>
<td>0.0705</td>
<td>0.0475</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.0380</td>
<td>0.2126</td>
<td>0.2094</td>
<td>0.2163</td>
<td>0.3237</td>
<td>0.0708</td>
<td>0.0479</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.0304</td>
<td>0.2117</td>
<td>0.2141</td>
<td>0.2217</td>
<td>0.3220</td>
<td>0.0711</td>
<td>0.0484</td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>0.0227</td>
<td>0.2108</td>
<td>0.2189</td>
<td>0.2274</td>
<td>0.3202</td>
<td>0.0714</td>
<td>0.0489</td>
<td></td>
</tr>
<tr>
<td>0.35</td>
<td>0.0147</td>
<td>0.2099</td>
<td>0.2236</td>
<td>0.2332</td>
<td>0.3184</td>
<td>0.0717</td>
<td>0.0494</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.0064</td>
<td>0.2089</td>
<td>0.2290</td>
<td>0.2392</td>
<td>0.3165</td>
<td>0.0721</td>
<td>0.0500</td>
<td>Maximum</td>
</tr>
</tbody>
</table>

4. Conclusions

In this paper, we describe the portfolio optimization methods based on the mean and the Value-at-Risk (VaR) under the Capital Asset Pricing Model (CAPM) with lagged, non-constant volatility and the long memory effect. We apply the method to analyze the return of five stock from Indonesia stock market. We obtain that the IHSG index follows the time series model ARFIMA(1, \( \tilde{d} \), 1)-GARCH(1,1) where the fractional difference constant is \( \tilde{d} = 0.3613183 \). These results have been used for the Mean-VaR portfolio optimization and it shows that an increment in the value of risk tolerance until \( r = 0.4 \) will lead to change in the portfolio composition weight, and expected return portfolios also increase until maximum of 0.0721. In above empirical example, the risk tolerances \( r > 0.4 \) does not provide feasible investments.

References