Single Premium of Equity-Linked with CRR and CIR Binomial Tree

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Abstract. Unit-linked life insurance is a life insurance product that is a hybrid because it provides two benefits at once, life insurance benefit and investment benefit. Equity-linked is a type of unit-linked life insurance that invest premiums paid partly in shares. Some of these insurance policies include options that give the right to the policyholder to terminate their policy contract and receive some cash. In this research, we study how to determine the amount of single premium endowment equity-linked using Cox, Rubenstein, and Ross (CRR) and Cox, Ingersoll, and Ross (CIR) binomial tree. CRR binomial tree is used to predict the stock price of several period ahead, while CIR binomial tree is used to predict the interest rate.

Keywords: equity-linked, single premium, binomial tree, stock price, interest rate.

INTRODUCTION

In the financial plans, investment and protection are two things that must be possessed. Unit-linked is a life insurance company product that combines these two functions. Unit-linked offers many investment choices, including stocks, bonds, money markets, and a mixture of stocks and bonds. Equity-linked is a unit-linked that invest at least 80\% premiums paid in shares. Some of equity-linked policies include an american put option that gives rights to policyholder to end the policy contract and receive some cash \cite{1}. This option is usually included the equity-linked endowment life insurance. Policyholder will get the benefit at maturity if he is still alive or when he dies before maturity during the option is not exercised \cite{2}.

In this paper, we will determine a single premium equity-linked with the binomial tree. Insurance premium which we pay once at the beginning of the policy contract should reflect how the future stock price and interest rate movements \cite{3}. Binomial tree is one of the methods used to predict them. Cox, Robenstein, and Ross (CRR) binomial tree method is used to predict stock price and Cox, Ingersoll, and Ross (CIR) is used to predict the future interest rate.

In the first section, some backgrounds of equity-linked are provided. In the next section, we explain the CIR binomial tree. The third section and the fourth section discuss the CRR binomial tree and equity-linked binomial tree respectively. And in the last section, we give the conclusion.
CIR BINOMIAL TREE

Cox, Ingersoll, and Ross (CIR) interest rate model was introduced in 1985 to address the Vasicek model that generates negative value in calculation of interest rate. The CIR model is shown by

\[ dr_t = \kappa (\theta - r_t) dt + \sigma r_t dW_t, \]

where \( \kappa, \theta, \sigma \) is unknown parameters that are positive and independent of time. \( W_t \) is a standard brownian motion. \( dr_t \) shows the interest rate changes. \( \kappa \) shows the average speed of interest rate changes. \( \theta \) and \( \sigma \) show the interest rate and volatility respectively [4].

If the life insurance policy entered into force upon signature (\( t=0 \)) until the maturity (\( T \)), the time interval \([0, T]\) can be divided into \( N \) time discrete, ie \([(i-1)h, ih], i=1,2,..., N\). The length of each discrete time period is \( h = \frac{T}{N} \). \( r_t \) is a interest rate value at time \( t \) and \( r_{t+1} \) is a interest rate value at the next period. The CIR model, which \( \sigma \sqrt{r_t} \) is not worth the constant and a function of \( r_t \), is not easy in its discretization, so we need the transformation of \( r_t \) first [5].

Choose \( x(r_t) \), the transformation of \( r_t \) that meets \( x(r_t) = \int_{r_t}^{1} \frac{1}{\sigma(Z_i)} dZ_i \). As a result

\[
\left( \sigma(r_t) \frac{\partial x(r_t)}{\partial r_t} \right) dW_t = dW_t \quad \text{and the volatility of } dx(r_t) \text{ is constant. } r_t \text{ binomial process can be obtained by } x(r_t) \text{'s invers.}
\]

Based on the relationship between \( r_t \) and \( x(r_t) \) where the \( r_t \)'s value cannot be negative with the result that the \( x(r_t) \) 's value should not be negative [5], the \( r_t \)'s value on binomial tree is determined as follows

\[
r_t = \begin{cases} 
\frac{1}{4} \left( x(r_t) \right)^2 \sigma^2, & x(r_t) > 0 \\
0, & \text{others}
\end{cases}
\]

Figure 1 and Figure 2 show the forward binomial tree of \( x(r_t) \) and \( r_t \) respectively.
The stock price model at time $t$ is

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where $\mu$ and $\sigma$ are unknown parameters and independent of time. As well as the CIR binomial tree, the time until to maturity interval $[0,T]$ is divided into $N$ discrete time steps $[(i-1)h, ih], i = 1, \ldots, N$, each of length $h = T/N$. The price of the stock one period ahead with probabilities of 0.5 will rise by

$$S_{i+1}^u = S_i \exp(\sigma \sqrt{h})$$

and decrease by

$$S_{i+1}^d = \frac{S_i}{\exp(\sigma \sqrt{h})},$$

where $S_i$ is the stock price at period/time $t \ (t \in [0,T])$ and $\sigma$ is the volatility of stock price [6]. Figure 3 shows the forward CRR binomial tree.
In this paper, it is assumed that stock price and interest rate are dependent or interconnected. In Kewal’s Theory, without any other factors that affect, they have a negative relationship [7]. The high interest rate will affect the present value of cash flows of the company, so that investment opportunities will not be as attractive anymore. The rising of interest rate also cause investors to move their investment in savings or deposits. Therefore, it can be formed a new binomial tree model which is a combination of interest rate and stock price binomial tree as follow:

![Figure 4. The Combination of Binomial Tree](image)

**EQUITY-LINKED BINOMIAL TREE**

This paper use endowment life insurance, which the policyholder will receive policy benefits if he died at the time t (0<t<T) or alive at time T. In addition, this policy benefits just can be received if the option did not exercised. The backward binomial tree of equity-linked can be formed based on the forward binomial tree of stock price and interest rate with the following conditions:

- In the case of death occurred between t and t+1, policy benefit $C_t^M = M$ and it is paid at time $t+1$.
- Option can be exercised at the beginning of each subintervals $t>0$. If the option is executed, the insurance company shall pay $R_t = R$ and $R \leq M$.
- If the policyholder is still alive until the beginning of period t and the option have not been exercised before, the policyholder has two alternatives:
  1. Exercise the option and it mean the policy contract expires.
  2. Continuing the contract and if he dies in the time interval $[t, t+1]$, the policy contract expires at $t+1$.
- If the policyholder is still alive until maturity, he will get benefit $C_T^v = f_T^v (F_T) = nS_T$

From the provisions above, the continuous value of policy contract is

$$W_t = h p_{x+ih} (e^{-rh}M) + h q_{x+ih} \left( e^{-rh} \left( 0.5V_{t+1}^u + 0.5V_{t+1}^d \right) \right)$$

and the contract value at time t is

$$V_t = \max (R, W_t)$$

where
The chance of the insured aged \( x + ih \) will survive until \( h \) years later.

The opportunity of insured aged \( x + ih \) will die within \( h \) years later.

\[ h P_{x+ih} \]

\[ h Q_{x+ih} \]

The risk-free interest rate.

\[ r \]

The value of \( V_{r+1} \) in the up position.

\[ V^u \]

A value of \( V_{r+1} \) in the down position.

\[ V^d \]

CONCLUSIONS

The equity-linked life insurance can be an alternative investment that it is attractive and promising because for the same time, an investor can protect the risks with the benefits of life insurance and also may invest in shares desired. Extra option rights on endowment life insurance (surrender option) will make this product more interesting because it will reduce investor losses due to falling share prices in the market. Besides that, the binomial tree can be an alternative in the fair premium life insurance calculation for easy interpretation and computation.

REFERENCES


