Valuation Of Reload Call Option With Binomial Tree

Yunita Wulan Sari¹, Gunardi²

¹Department of Mathematics, Universitas Gadjah Mada, email : yunita-ws@ugm.ac.id
²Department of Mathematics, Universitas Gadjah Mada

Abstract. Along with the rapid growth of investment and finance, the investors compete with each other to find a way to maximize profits and minimize losses. In option trading, one of the ways used is adding a reload feature on the option. Reload option is option that give the right to its holder to exercise the option, then renew it at some time prior to maturity. In this research, we study how to determine the pricing of reload call option with the Black-Scholes stock price model and the binomial tree, then applied to the stock of Telekomunikasi Indonesia (TLKM.JK). From this study, we obtain the valuation of reload call option which is simple and easy in interpretation. In addition we can also conclude that if the time to maturity is longer, the price of reload call option is more expensive and if the exercise price is higher, the price of reload call option is cheaper.

2010 Mathematics Subject Classification: 62P05

Keywords: option, reload option, reload call option, Black-Scholes stock price model.

Section: SS-07.

INTRODUCTION

Along with the rapid growth of investment and finance, the investors compete with each other to find a way to maximize profits and minimize losses. Option is one of well known alternative investments. Traded option have added some features to make it more attractive and desirable. One of the famous features is reload. Option with reload feature added is called reload option (also called restoration option, replacement option, or accelerated ownership option). Reload option gives the owner the right to execute his option and update it at one or several times before maturity (0<T₁<T). The number of new options granted is equal to the number of shares needed to pay the strike price using the stock at its current value [1].

Huddart et al (1999) added some modification to the algorithm of reload option price valuation, then applied the binomial option pricing model on its study case. Further, he get
the result that option price valuation with binomial approach is easy on interpretation. Besides that, it can be shown that the price of reload option is higher than the common option [2]. Although reload option typically involve exercise at many dates, the optimal exercise policy is simple (always exercise when in the money) and surprisingly robust to the assumptions about the underlying stock price and dividend process. As a result, we obtain general reload option valuation formulas that can be evaluated numerically using integral as a formal valuation. Furthermore, under the Black-Scholes assumptions with or without continuous dividends, there are even simpler formulas for prices and hedge ratios [3]. Whereas, the main factor that determine the reload option valuation is the underlying asset price [4].

Reload feature added on option make the difficult and complex option valuation. This time, the reload option is unstudied and sold in Indonesia. In this research, we will studied how to determine the value of reload option with the Black-Scholes model and binomial tree method, then applied it on Indonesian stock data.

**CONTRACT STRUCTURE OF RELOAD CALL OPTION**

Reload option is designed to can be executed before maturity and automatically grants new option. With a regular reload, the number of new options granted is equal to the number of shares needed to pay the strike price using the stock at its current value. If reload only done once before maturity, the value of option at reloaded time \( T_i \) \((0<T_i<T)\) is

\[
V_{T_i} = \max \left( S_{T_i} - K, 0 \right)
\]

and the value of reload option at maturity is

\[
V_T = \begin{cases} 
\frac{K}{S_{T_i}} \max \left( S_T - S_{T_i}, 0 \right), & \text{if } S_{T_i} > K \\
\max \left( S_T - K, 0 \right), & \text{if } S_{T_i} < K 
\end{cases}
\]

where

- \( K \) : strike price
- \( S_{T_i} \) : market stock price at reload time \( T_i \)
- \( S_T \) : market stock price at maturity.

In general, the contract structure of reload call option with \( k \) execution/reload time and the order of strike price on each reload time \((K, S_1, S_2, \ldots)\) is as follows :
At the time the contract was agreed (t=0), the option buyer has one option and no share.

At the first reload, the option buyer has \( \frac{K}{S_1} \) new reload options and \( 1 - \frac{K}{S_1} \) shares.

At the second reload, the option buyer has \( \frac{K}{S_1} \times \frac{S_1}{S_2} = \frac{K}{S_2} \) new reload options and

\[
\left( 1 - \frac{K}{S_1} \right) + \left( \frac{K}{S_1} \times \left( 1 - \frac{S_1}{S_2} \right) \right) = \left( 1 - \frac{K}{S_2} \right) \text{ shares.}
\]

And at the k-th reload, the option buyer will have \( \frac{K}{S_k} \) new reload options and \( 1 - \frac{K}{S_k} \) shares.

**STOCK PRICE BINOMIAL TREE**

If the option is valid from the time signed (t=0) until the maturity (T), the time interval \([0,T]\) can be divided into N discrete time, ie \([i-1]h, ih\], \(i = 1, 2, ..., N\). Each discrete time period length is \( h = \frac{T}{N} \). In fact, the market stock price always turned up or down in line with changing times. The possibility of two-way change is used as the basic of binomial models. For example, the stock price at t=0 is \( S_0 \) and it will be up to \( S_{i+1}^u \) and down to \( S_{i+1}^d \) with probability are \( p \) and \( (1-p) \) respectively at \( t+1 \).

In the case of a constant interest rate, we assume the following risk-neutralized dynamics for stock prices, \( S \):

\[
d \ln (S_t) = \left( \mu_s - \frac{1}{2} \sigma_s^2 \right) dt + \sigma_s dW_t
\]

\( \mu_s \) and \( \sigma_s \) are unknown parameters. They are independent of time. These two of parameters can be estimated by historical stock price. \( W_t \) is standard brownian motion [5]. Based on this stock price model, each discrete time period whose its length is \( h \) applies

\[
\ln \left( S_{i+1}^u \right) = \ln (S_i) + \sigma_s \sqrt{h} \quad \text{and} \quad \ln \left( S_{i+1}^d \right) = \ln (S_i) - \sigma_s \sqrt{h} \, .
\]

Therefore, it can be defined stock price at time \( t+1 \) if the stock price at time \( t \) is known as

\[
S_{i+1}^u = S_i e^{\sigma_s \sqrt{h}}
\]

and
Figure 1 shows the stock price binomial tree at each node.

![Stock Price Binomial Tree](image)

**RELOAD CALL OPTION BINOMIAL TREE**

Based on the stock price binomial tree, it can be formed the reload call option. The notation defined is as follows:

- $S_{i,j}$: stock price at node $(i,j)$
- $R_{i,j,k}$: $k$-th value of the strike price at node $(i,j)$
- $V_{i,j,k}$: $k$-th value of the reload call option at node $(i,j)$
- $r$: risk-free interest rate (assume constant)

The following are the strike price setting at each node, and we divide this setting into two situations:

**Situation 1**: $S_{i,j} > R_{i,j,k}$ (in the money)

Option holder has two alternative, there are exercise the option and not. $S_{i,j}$ and $R_{i,j,k}$ will sent to $R_{i+1,j+1}$ and $R_{i+1,j}$.

**Situation 2**: $S_{i,j} \leq R_{i,j,k}$ (out the money or at the money)

Option holder will not exercise his option, $R_{i,j}$ will sent to $R_{i+1,j+1}$ and $R_{i+1,j}$.

Define $V_e$ and $V_h$ as the early exercise value and the holding value respectively. When
i<n, we explain the approach which we calculate $V_{i,j,k}$ as follows:

- The value of reload call option at maturity is
  
  $$V_{n,j,k} = \frac{K}{R_{n,j,k}} \max \left( S_{n,j} - R_{n,j,k}, 0 \right), \text{ for all } k$$

- The value of reload call option at node (i,j) is

  **Situation 1 :** $S_{i,j} > R_{i,j,k}$
  
  $$V_{i,j,k} = \max \left( V_e, V_h \right)$$

  where
  
  $$V_e = \frac{K}{R_{i,j,k}} \max \left( S_{i,j} - R_{i,j,k}, 0 \right) + e^{-rh} \left( pV_{i+1,j+1,z} + (1-p)V_{i+1,j,z} \right)$$

  $z$ represents the $z$-th value of $R_{i+1,j+1}$ or $R_{i+1,j}$ which equals to $S_{i,j}$.

  $$V_h = e^{-rh} \left( pV_{i+1,j+1,y} + (1-p)V_{i+1,j,y} \right)$$

  $y$ represents the $y$-th value of $R_{i+1,j+1}$ or $R_{i+1,j}$ which equals to $R_{i,j,k}$.

  **Situation 2 :** $S_{i,j} \leq R_{i,j,k}$

  $$V_{i,j,k} = e^{-rh} \left( pV_{i+1,j+1,y} + (1-p)V_{i+1,j,y} \right)$$

  where $y$ represents the $y$-th value of $R_{i+1,j+1}$ and $R_{i+1,j}$ which equals to $R_{i,j,k}$.

  $V_{0,0,1}$ is the current reload call option.

**RELOAD CALL OPTION OF TLKM.JK**

In study case, we will calculate the value of reload call option at beginning of August 2015 and the maturity date is three month. It is assumed that option just can be reload at the beginning of month until maturity. Based on BI rate [6], risk-free interest rate paid per month is 0.6045%. TLKM.JK’s stock prices at January 2014-August 2015 are used to predict the future stock prices [7].

Figure 2 shows stock price, strike price, and the value of reload call option at each node. The TLKM.JK’s value of reload call option is IDR 161,994 per share.
Figure 2. Reload Call Option Binomial Tree

Figure 3. Reload Call Option of TLKM.JK in Several Maturity Date
Figure 3 explain that the length of time to maturity gives a positive influence to the value of reload call option. However, the strike price gives a negative influence (Figure 4).

REFERENCES