PROCEEDINGS of
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Preface
from President of IndoMS for Proceeding of IICMA 2009:

First of all, I would like to pray for God for His mercy so that we could finish the Proceeding of IICMA 2009 (IndoMS International Conference on Mathematics and its Applications 2009) held on October 12th-13th, 2009 at the Departement of Mathematics Gadjah Mada University Yogyakarta Indonesia. On behalf of the IndoMS (Indonesian Mathematical Society), I would like to say congratulation to all authors in the proceeding.

IndoMS or formerly known as "Himpunan Matematika Indonesia" is a forum for mathematicians and users of mathematics as well as people who have interest in enhancing mathematics in Indonesia. The Society is a scientific, nonprofit, non-governmental and professional organization. It was established on July 15th, 1976 in Bandung, West Java. The objectives of the Society are to enhance and extend mathematical knowledge, extend education in the Mathematical sciences, and to increase the role of mathematics in Indonesia. In 2009 IndoMS has 1.151 members consisting of university teachers, mathematicians, statisticians and mathematics-education researchers from 30 Indonesian universities, and school teachers from elementary and high schools. IndoMS has established 8 provincial officers to stimulate and enhance mathematical activities in the country. The branches are branch Special Territory of Yogyakarta and Central Java, branch Banten, Special Territory of Jakarta, and West Java, branch East Java, branch South and West Sulawesi, branch South Kalimantan, branch South Sumatera, branch Nanggroe Aceh Darussalam and South Sumatra, and branch East Nusa Tenggara. Since 1976, IndoMS has already 14 times organized National Conference in Mathematics and National Congress. The next National Conference in Mathematics and National Congress will be held in Manado State University, North Sulawesi on June 30–July 3, 2010. Since 2006, IndoMS also has already 3 times organized National Conference in Mathematics Education. The next National Conference in Mathematics Education will be held at the Yogyakarta State University in 2011.

Starting in 2009 IndoMS organize International Conferences. IICMA2009 is IndoMS International Conference on Mathematics and its Applications 2009. It is majority supported by Directorate General of Higher Education (DGHE), Department of National Education, Indonesia through "Professional Organization Symposium Competition Program" (Program Hibah Symposium Organisasi Profesi). IndoMS is one of professional organizations which granted by this program. In this conference we facilitate researchers and users of mathematics to exchange ideas and discuss research results and development of mathematics internationally in the fields of mathematics including mathematics education and its applications.

All 166 full papers in the conference has been reviewed by 36 competence experts. As results of the conference, we got 20 papers are feasible to be published in international journals, 17 papers are feasible to be published in aspirated international journals, 32 papers are feasible to be published in national journals, 65 papers published in this proceeding, and 32 papers are rejected.

Finally, I would like to express my sincere appreciation to:
- Dean of Faculty of Mathematics and natural Sciences and Rector of Gadjah Mada University for the permission and cooperation in holding the Conference
- Steering Committee and Organizing Committee for all efforts for the success of the conference
- All invited speakers and all participants from Indonesia and abroad for the active participation of the conference.
- All editors for all efforts to finish this proceeding.
  Last but not least, my sincere appreciation is also extended to the DGHE for the major support of the conference.

Yogyakarta, January, 2010
President of IndoMS 2008-2010

Prof. Dr.rer.nat. Widodo
Preface from the Committee

The proceeding of IICMA Conference is a collection of all selected papers that were presented in IndoMS International Conference on Mathematics and its Applications (IICMA) 2009 held at Department of Mathematics – Gadjah Mada University, Yogyakarta, October 12th – 13th, 2009. The selected papers are based on the reviewed results by 36 competence reviewers. Each paper has been reviewed by at least two reviewers. On behalf of the Committee, we would like to say thank you very much to all of the reviewers.

There are 118 papers in this proceeding coming from diverse aspects of mathematics ranging from Analysis, Applied Mathematics, Algebra, Theoretical Computer Science, Mathematics Education, and other related topics. We are sure the papers will inspire, not only writers, but also many other researches in developing mathematics and its applications. Please find the benefit of the proceeding.

Yogyakarta, January 2010
On behalf of the Committee IICMA 2009

[Signature]

Dr. Ch. Rini Indrati, M.Si.
Chair
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FEEDBACK ZERO-SUM LINEAR QUADRATIC
DYNAMIC GAME FOR DESCRIPTOR SYSTEM

SALMAH

Abstract. In this paper we present necessary and sufficient conditions for existence of Nash equilibrium of linear quadratic continuous zero-sum two player dynamic games for index one descriptor system. We assume that we give a linear feedback to the game. The connection of the game solution with solution of N couple Riccati equation will be studied.

Key words and Phrases : Nash equilibrium, feedback, zero-sum, linear quadratic dynamic game, descriptor system, index one

1. Introduction

In the last decade, there has been increasing interest to study the problem in economics with dynamic game approach. Particularly, in area of environmental economics and macro-economics policy coordination, it is natural to model the problems as dynamic game[1], [7] and [24]. With this approach, the effect of the execution control strategy of the game to dynamic of the model can be analyzed([2], [5], [8], [9], and [10]). In applications one often encounters systems described by differential equations system subject to algebraic constraints. The descriptor systems, gives a realistic model for this systems ([3], [4], [11], [12], [13], [14], [15] and [16]).

In policy coordination problems, questions arise, are policies coordinated and which information do the parties have. One scenario is feedback Nash. According this, the parties can react to each other’s policies, therefore it has large economic relevance.

In this paper we will consider a linear feedback zero-sum dynamic game in which the player satisfy a linear descriptor system and minimize quadratic objective function. For finite horizon problem, solution of generalized Riccati differential equation is studied. If the planning horizon is extended to infinity the differential Riccati equation will become an algebraic Riccati equation. Particular attention will be given to computational aspect of the problem.

The purpose of this paper is to extend the investigation by Salmah et.al. in [18], [19], [20], [21], [22], [23] and Engwerda et.al in [16] where the game is non-zero-sum and the scenario for the game is open-loop. In this paper the non-zero-sum game with include linear feedback strategy will be applied to zero-sum game with feedback strategy.

Until recently, except for the work of the writer and colleague, a study of
differential game for descriptor system is lacking. Such first step studies have been carried out with assumption that the game is open loop. To study necessary and sufficient condition for existence of Nash solution of the game Hamiltonian method was used as in [18]. To find the optimal solution of the dynamic game for descriptor system with a finite planning horizon, the problem is related to the solution of differential Riccati equation. The differential Riccati equation is a generalization and combination of differential Riccati equation for linear quadratic dynamic game with 'ordinary system' and differential Riccati equation for linear quadratic optimal control for descriptor system in [19] and [20]. The work of linear quadratic dynamic game for descriptor system with infinite horizon case and studied algebraic Riccati equation for the game is in [21]. A simplifying assumption can be made, namely descriptor system with index one [6].

In those studies, the assumption for the game is open-loop. Open-loop game is a benchmark to study more complicated game. This strategy is based on assumption that the parties act non-cooperatively and the only information they have is the present state and the model structure. In this scenario the parties can not react each other. Therefore its economic relevance is limited.

In this paper we will study the game that including feedback Nash, in which the parties can react to each other’s policies. This scenario has large economic relevance. The result from non-zero-sum game with linear feedback strategy will be applied to zero-sum game with linear feedback strategy.

2. Preliminaries

Linear quadratic dynamic game can be considered as a combination of linear quadratic optimal control and game theory. In linear quadratic dynamic game, N parties (called players) try to minimize their individual quadratic objective function and give control to 'ordinary' state space system.

Although it has many applications, ordinary linear quadratic optimal control, often does not provide a physical meaning in controlling, because the state variable does not corresponds with variable that we want to control. Descriptor systems have great capacity for system modeling because they can preserve structure of physical system and can include nondynamic mode and impulsive mode. Therefore they have a potential applicability for a wide class of systems. Descriptor system described by a set of ordinary equations subject to some algebraic constraints.

In linear quadratic dynamic game for descriptor system we consider the problem of two players who like to optimize their quadratic cost function performance depending both on the state and control variables. The system is described by a set of differential and algebraic equations which is called a descriptor system. The game with two players can be expressed mathematically, that the players give control to descriptor system

\[ E\dot{x} = Ax + B_1 u_1 + B_2 u_2, \quad E x(0) = E x_0. \]  

with \( E \in \mathbb{R}^{n \times n}, A \in \mathbb{R}^{n \times n}, B_1 \in \mathbb{R}^{n \times m_1}, B_2 \in \mathbb{R}^{n \times m_2}, \) \( x(t) \) descriptor vector n dimension. While \( u_i(t), i=1, ..., n \) are control vector \( m_i \) dimension which is done by i-th player, \( i=1, ..., n \). Matrix E generally singular with rank \( E = r < n \). The players minimizing objective functions in the Nash sense of the form
\[ J_i(u_i, u_2) = \frac{1}{2} x(T)^T E_i^T K_{fi} Ex(T) + \frac{1}{2} \int_0^T \left( x(t)^T Q_i x(t) + u_i^T(t) R_{fi} u_i(t) + u_2^T R_{fi} u_2(t) \right) dt, \]

\( i = 1, 2 \) \hspace{1cm} (2)

with all matrices symmetric. Furthermore \( Q_i \) and \( K_{fi} \) semi positive definite and \( R_{fi} \) positive definite.

In this paper we will consider a linear feedback strategy of the linear quadratic dynamic game for descriptor system. Below is definition of feedback strategy.

**Definition 2.1.** The set of control actions \( F^* = (F_1^*, F_2^*) \) is called a feedback Nash equilibrium if for all \( i = 1, 2 \) \( J_i(x_0, F_i^*) \leq J_i(x_0, F_i^*(\alpha)) \) for every consistent \( x_0 \) and for each matrix \( \alpha \) such that \( F_i^*(\alpha) \in F_{fit}^* \).

Under some assumptions such as regularity, impulse controllability and index one we will solve the game, both for finite and in finite planning horizon. To find solution of linear quadratic dynamic game for descriptor system with finite horizon case, a differential Riccati will be derived. The relationship between the existence of solution of differential Riccati equation and solution of the game will be considered. For infinite horizon case algebraic Riccati equation that associated with the game will be studied.

Now we will initiate the zero-sum game. Consider the problem that the players satisfy (1). The zero-sum game is a game with the player one minimizing objective functions in the Nash sense of the form

\[ J_1(u_1, u_2) = \frac{1}{2} x(T)^T E_i^T K_{fi} Ex(T) + \frac{1}{2} \int_0^T \left( x(t)^T Q_i x(t) + u_1^T(t) R_{fi} u_1(t) - u_2^T(t) R_{fi} u_2(t) \right) dt, \]

\( i = 1, 2 \) \hspace{1cm} (3)

For the player two, the opposite objective function

\[ J_2(u_1, u_2) = -J_1(u_1, u_2) \]

\( i = 1, 2 \) \hspace{1cm} (4)

with all matrices symmetric. Furthermore \( Q_i \) and \( K_{fi} \) semi positive definite and \( R_{fi} \) positive definite.

Assumption which is needed will be given.

**Assumption 2.1.** Descriptor system (1) regular, impulse controllable and finite dynamic stabilizable which satisfy

\( i) \quad |sE - A| \neq 0, \forall s \neq 0, \) except for a finite number of \( s \in \mathbb{R}, \)

\( ii) \quad \text{Im} E + \text{Im} A(\ker E) + \text{Im} \left( B_1 \mid B_2 \mid \cdots \mid B_N \right) = \mathbb{R}^n, \)

\( iii) \quad \text{rank} \left( sE - A \mid B_1 \mid B_2 \mid \cdots \mid B_N \right) = n \quad \forall s, \text{Re}[s] \geq 0. \)
Recall from [12] for optimal control problem with descriptor system we will consider the descriptor system
\[
\begin{pmatrix}
E & 0 & 0 \\
0 & E^T & 0 \\
0 & 0 & 0 
\end{pmatrix}
\begin{pmatrix}
\dot{x}(t) \\
\dot{y}(t) \\
\dot{u}(t) 
\end{pmatrix} =
\begin{pmatrix}
A & 0 & B \\
-\tilde{Q} & -A^T & -BR^1B^T \\
BR^1B^T & B^T & R 
\end{pmatrix}
\begin{pmatrix}
x(t) \\
y(t) \\
u(t) 
\end{pmatrix},
\]
with \( \gamma \) is Lagrange multiplier in Hamiltonian function. For optimal control problem with descriptor system we need the following assumption.

**Assumption 2.2:** The Descriptor system is regular and impulse free i.e
\[
\text{Im} \tilde{E} + \text{Im} \tilde{A} (\ker \tilde{E}) = 9k^{2n+m},
\]
where
\[
\tilde{E} =
\begin{pmatrix}
E & 0 & 0 \\
0 & E^T & 0 \\
0 & 0 & 0 
\end{pmatrix},
\tilde{A} =
\begin{pmatrix}
A & 0 & B \\
-\tilde{Q} & -A^T & -BR^1B^T \\
BR^1B^T & B^T & R 
\end{pmatrix}
\]

If Assumption 2.2 is satisfied then the controlled system will be regular and impulse controllable.

### 3. The Finite Planning Horizon

In this section we consider the zero-sum game that the player give control to system (1) and the playery to minimize in the Nash sense (3) and (4) under the assumption that \( T \) is finite. For that purpose we first need to consider the non-zero-sum game in which the players give control to system (1) and try to minimize (2). For non-zero-sum game with linear feedback strategy the differential Riccati equation
\[
E^T \dot{K}_1 + (A - S_2K_2)\dot{K}_1 + L_1(A - S_2K_2 - L_2S_2K_2) - K_2S_2K_2 + Q_1 = 0,
\]
\[
E^T \dot{K}_2 + (A - S_1K_1)\dot{K}_2 + L_2(A - S_1K_1 - L_2S_2K_2 - K_1S_2K_1) + Q_2 = 0,
\]
with
\[
L_1E = E^T K_1, \quad L_2E = E^T K_2,
\]
\[
S_1 = B_1R_{11}^{-1}B_1^T, \quad S_2 = B_2R_{21}^{-1}B_2^T, \quad S_{21} = B_2R_{21}^{-1}R_{12}R_{12}^{-1}B_2^T, \quad S_{12} = B_1R_{11}^{-1}R_{21}R_{21}^{-1}B_1^T,
\]
play a crucial role. Theorem below give relationship between solution of differential Riccati equation (5) and solution of non-zero-sum linear quadratic game with descriptor system that include feedback strategy.

**Theorem 3.1.** The two player non-zero-sum linear quadratic differential game with descriptor system (1), (2) has, for every consistent initial state, a linear feedback Nash equilibrium if and only if the set of differential Riccati equation (5) has a set of symmetric solutions \( K_1, K_2, L_1, L_2 \) on \([0,T]\).
Proof: Assume \( u_i^*(t) = F_i^*(t)x(t) \), \( t \in [0, T] \), \( i = 1, 2 \), is a set of linear feedback equilibrium actions. Then according to the definition of feedback equilibrium, the following linear quadratic regulator problem has a solution \( u_i^*(t) = F_i^*(t)x(t) \), for all \( x_0 \) subject to the system

\[
Ex(t) = (A + B_1F_1^*(t))\dot{x}(t) + B_1u_1(t), \quad Ex(0) = E\tilde{x}_0.
\]

According to [12], this regulator problem has a solution if the Riccati differential equation

\[
E^T\dot{K}_1 = (A - B_2F_2^*(t))^TK_1(t) + L_1(t)(A + B_2F_2^*(t)) - L_1(t)SK_1(t) = (Q_1 + F_2^*(t)R_2F_2^*(t))
\]

has a symmetric solution \( K_1(.) \) on \([0, T]\). Moreover, the solution for this optimization problem is given by

\[
u_i^*(t) = -R_1^{-1}B_1^TK_1(t)x(t).
\]

For the second player the proof is analog.

Now we will proof the converse part of the theorem. Assume we choose the feedback strategy \( F_1 = -R_1^{-1}B_1^TK_1(t)x(t) \), \( F_2 = -R_2^{-1}B_2^TK_2(t)x(t) \), with \( K_1(t) \) and \( K_2(t) \) is solution of differential Riccati equation (3.1). Define

\[
\gamma_1(t) = K_1(t)x(t), \quad \gamma_2(t) = K_2(t)x(t).
\]

Define the equations to \( t \) we have

\[
E^T\dot{\gamma}_1(t) = E^T\dot{K}_1(t)x(t) + E^TK_1(t)\dot{x}(t),
\]

\[
E^T\dot{\gamma}_2(t) = E^T\dot{K}_2(t)x(t) + E^TK_2(t)\dot{x}(t).
\]

Based on equation of the system (2.1) we have

\[
Ex(t) = Ax(t) - B_1R_1^{-1}B_1^TK_1(t)x(t) - B_2R_2^{-1}B_2^TK_2(t)x(t),
\]

or

\[
Ex(t) = Ax(t) - B_1R_1^{-1}B_1^TK_1(t)x(t) - B_2R_2^{-1}B_2^TK_2(t)x(t).
\]

Based on Riccati differential equation (3.1) we have

\[
E^T\dot{K}_1 = -(A - S_2K_2)^TK_1 - L_1(A - S_2K_2) - Q_1 + L_1S_1K_1 + K_2S_{11}K_2,
\]

\[
E^T\dot{K}_2 = -(A - S_1K_1)^TK_2 - L_2(A - S_1K_1) - Q_2 + L_2S_2K_2 + K_1S_{12}K_1.
\]

Therefore we have

\[
E^T\dot{\gamma}_1(t) = -(A - S_2K_2(t))\gamma_2(t) - Q_1\gamma_1(t) + K_2(t)S_{11}K_2(t)x(t).
\]

Based on [23] and the definition of feedback Nash equilibrium it complete the proof.

Applying Theorem 3.1 to zero-sum game descriptor system we will find the following theorem.

**Theorem 3.2.** The two player zero-sum linear quadratic differential game with descriptor system (1), (3) and (4) has, for every constant initial state, a linear feedback Nash equilibrium if and only if the set of differential Riccati equation
\[ E^T \dot{K} + A^T K + LA - L(S_1 - S_2)K + Q = 0 \]  

has a set of solutions \( K, L \) on \( [0, T] \).

**Proof:** According to Theorem 3.1, the corresponding generalized differential Riccati equation will have form

\[ E^T \dot{K}_1 + (A - S_2 K_2)^T K_1 + L_1 (A - S_2 K_2) + Q - L_1 S_2 K_2 - K_1 S_1 K_2 = 0, \]  

\[ E^T \dot{K}_2 + (A - S_1 K_1)^T K_2 + L_2 (A - S_1 K_1) - Q - L_2 S_1 K_1 - K_2 S_2 K_1 = 0 \]

Adding (7) and (8) give the following differential equation

\[ E^T (\dot{K}_1 + \dot{K}_2) + (A - S_1 K_1 - S_2 K_2)^T (K_1 + K_2) + (L_1 + L_2) (A - S_1 K_1 - S_2 K_2) = 0 \]

Obviously \((K_1 + K_2)(.) = 0\) and \((L_1 + L_2) (.) = 0\) satisfy this equation. Since the solution of this differential equation is unique we have that \( K_1 = -K_2 \) and \( L_1 = -L_2 \). Substitute this into (7) we get (6).

### 4. The Infinite Planning Horizon

In this section we consider the zero-sum game that the player satisfy (1) and the first player try to minimize the cost function

\[ J_1(u_1, u_2) = \frac{1}{2} \int_0^T \left( x(t)^T Q x(t) + u_1^T(t) R_1 u_1(t) + u_2^T(t) R_2 u_2(t) \right) dt, \]  

(9) with all matrices symmetric. Furthermore \( Q_1 \) and \( K_{1T} \) semi positive definite and \( R_{1y} \) positive definite. (9) with all matrices symmetric. Furthermore \( Q_1 \) and \( K_{1y} \) semi positive definite and \( R_{1y} \) positive definite. For that purpose we first need to consider the non-zero-sum game in which the player give control to system (1) and they try to minimize the cost function

\[ J_1(u_1, u_2) = \frac{1}{2} \int_0^T \left( x(t)^T Q x(t) + u_1^T(t) R_1 u_1(t) + u_2^T(t) R_2 u_2(t) \right) dt, \]  

(11) In infinite planning horizon case it can be prove that the differential Riccati equation become an algebraic Riccati equation, the solution become constant, and the differential term become zero. Therefore now we consider the algebraic Riccati equation

\[ (A - S_2 K_2)^T K_1 + L_1 (A - S_2 K_2) - L_1 S_2 K_2 - K_1 S_1 K_2 + Q_1 = 0, \]

\[ (A - S_1 K_1)^T K_2 + L_2 (A - S_1 K_1) - L_2 S_1 K_2 - K_2 S_2 K_1 + Q_2 = 0, \]

\[ L_1 E = E^T K_1, \]

\[ L_2 E = E^T K_2, \]  

(12)
with \( S_1 = B_1R_1^{-1}B_1^T \), \( S_2 = B_2R_2^{-1}B_2^T \), \( S_{12} = B_1R_1^{-1}R_{12}R_2^{-1}B_2^T \).

For non-zero-sum infinite horizon case we have the following theorem.

**Theorem 4.1.** The two player non-zero-sum linear quadratic differential game with descriptor system (1), (10) has, for every consistent initial state, a linear feedback Nash equilibrium if and only if the set of algebraic Riccati equation (12) has a set of solutions \( K_1, K_2, L_1, L_2 \) on \([0,T]\).

Apply the result of Theorem 4.1 to zero-sum infinite horizon game we get the following theorem.

**Theorem 4.2.** The two player linear quadratic differential game with descriptor system (1), (11) has, for every consistent initial state, a linear feedback Nash equilibrium if and only if the set of differential Riccati equation

\[
\dot{K} + LA - L(S_1 - S_2)K + Q = 0
\]

with \( LE = E^T K \), has a set of solutions \( K, L \) on \([0,T]\).

**Proof:** According to Theorem 4.1, the corresponding generalized differential Riccati equation will have form

\[
(A - S_1K_1 - S_2K_2)K_1 + L_1(A - S_1K_1) + Q - L_1S_1K_1 - K_1S_1K_2 = 0,
\]

\[
(A - S_1K_1 - S_2K_2)K_2 + L_2(A - S_1K_1) - Q - L_2S_1K_1 - K_2S_1K_2 = 0
\]

Adding (14) and (15) give the following algebraic equation

\[
(A - S_1K_1 - S_2K_2)^T(K_1 + K_2) + (L_1 + L_2)(A - S_1K_1 - S_2K_2) = 0
\]

Obviously \((K_1 + K_2)v = 0\) and \((L_1 + L_2)v = 0\) satisfy this equation. Since the solution of this differential equation is unique we have that \( K_1 = -K_2 \) and \( L_1 = -L_2 \). Substitute this into (14) we get (13).

Based on [12] we can find solution of the algebraic Riccati equation by defining generalized eigenvalue problem that need further investigation.

### 3. Concluding Remarks

This paper consider 2 player zero-sum linear quadratic dynamic game with descriptor systems for finite horizon and infinite horizon case with linear feedback Nash equilibrium. The paper consider 2 couple Riccati-type differential equation for finite horizon case and algebraic Riccati equation for infinite horizon case. We derive theorem that consider relationship between solution of the Riccati equation and solution of the game.
References


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